

12.1 Three Dimensional Coordinate System.

Mathematical Modeling of

- a) Physical line.
- b) " Plane
- c) Space

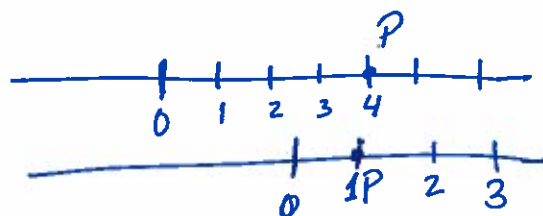
a) Line

Geometrical representation

Pick a ^{physical} point P in the line



This is a unique point, but it can be mathematically represented in many different ways



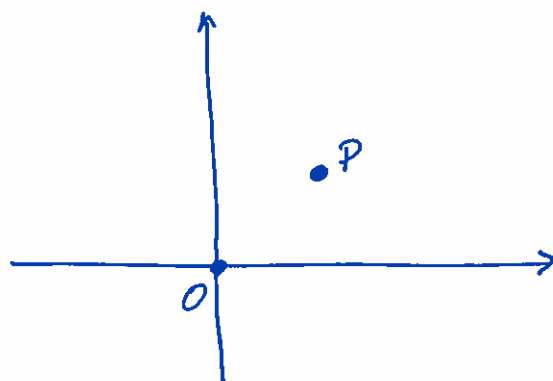
$x=4$ x is called coordinate of P .
 $x=1$

It depends where the "0" "origin of coordinates" is chosen and it also depends on the scale chosen (unit of measure).

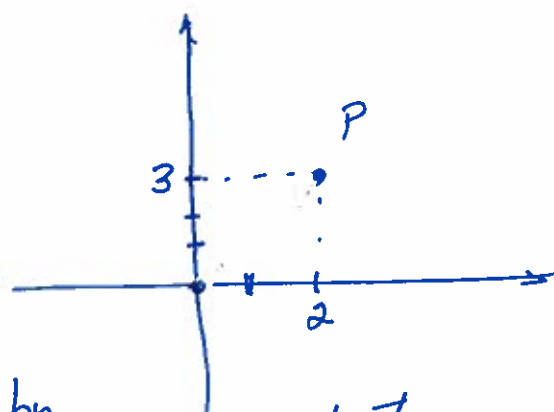
For example miles vs kms.

So representation: $P \rightarrow x=4$ (For our first choice of origin, unit, and orientation)

- b) Plane. Consider one of the classroom walls.
Pick a point



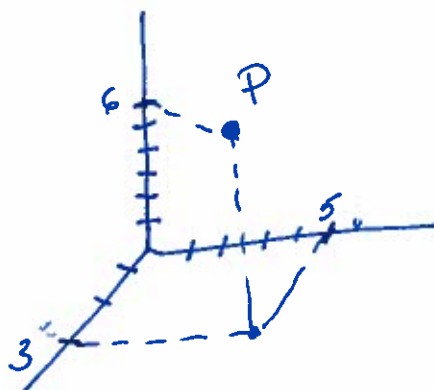
- b₁) Choose an "origin" and draw two perpendicular "axes"
b₂) " a unit of measure and an orientation "+", "-".



Physical point
is represented by
P $(2, 3)$
 $\rightarrow$ x-coordinate
 $\rightarrow$ y-coordinate

c) Point in space

P $\rightarrow (x, y, z)$ "ordered triple"
 $\downarrow \quad \downarrow \quad \downarrow$
 x-coord y-coord z-coord



Definition :

$$\mathbb{R}^3: \text{Cartesian product} = \mathbb{R} \times \mathbb{R} \times \mathbb{R} \\ = \left\{ (x, y, z): x, y, z \overset{\uparrow \text{in}}{\in} \mathbb{R} \right\}$$

One-to-One correspondence

$$\begin{array}{c} \text{Points} \\ P \\ \text{in space} \end{array} \longrightarrow (a, b, c) \overset{\uparrow \text{in}}{\in} \mathbb{R}^3$$

We also call P as $P(a, b, c)$

12.1 Three-Dimensional Coordinate Systems

Motivation

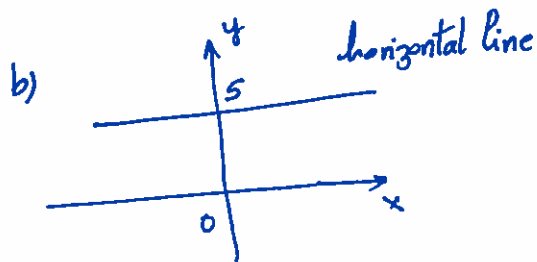
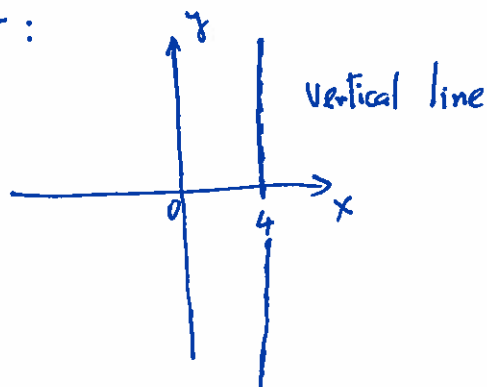
1.- What does the following equations represent in the plane? x - y plane

a) $x = 4$, b) $y = 5$

c) $x^2 + y^2 - 2x + 2y = \frac{1}{4}$

Answer:

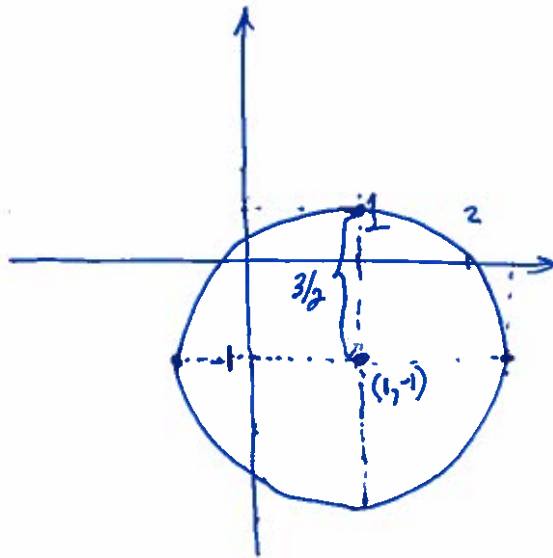
a)



c) $(x-1)^2 + (y+1)^2 = \frac{1}{4} + 1 + 1 = \frac{9}{4}$

or $(x-1)^2 + (y+1)^2 = \left(\frac{3}{2}\right)^2$

Graph ?



Circle with origin at $C(1, -1)$ and radius $r = \frac{3}{2}$.

Definition of a circle: ?

Concepts :

i) Distance between two points:

$P_1(x_1, y_1)$ and $P_2(x_2, y_2)$.

• $P_2(x_2, y_2)$

• $P_1(x_1, y_1)$

ii) Definition of a Circle:

"A circle is a set of all points whose distance from a fixed point $C(h, k)$ is r ."

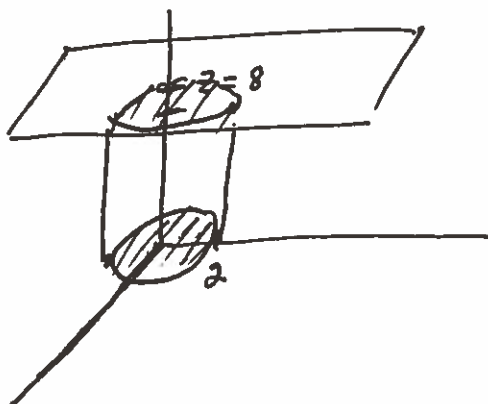
Equation of a Circle:

$$(x-h)^2 + (y-k)^2 = r^2.$$

3-D

Notions of:

- Physical points
- Coordinates $P_1(x_1, y_1, z_1)$, $P_2(x_2, y_2, z_2)$.
- Coordinate planes.
- What does
 - a) $x=4$ b) $y=5$
 represent in 3-D?
- Distance $|P_1 P_2|$ for $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$
- Equ. of a sphere. Definition of Sphere of radius r .

Probl. #Solid cyl. below $z=8$ on or above disk in xy -plane of radius 2.
Center at the origin.

$$x^2 + y^2 \leq 4$$

$$0 \leq z \leq 8$$